# Fundamental Study of Analytical Techniques for Predicting Thermal Strain in Conductors

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**Abstract**

**Copper materials are widely used in industrial equipment and electric vehicles that support infrastructure. Copper materials have the advantage of low electrical resistance and small electrical loss. Since they are soft and have low strength, accuracy is required for life calculations.**

**This led us to conduct research on the relationship between stress and strain that takes into account various usage conditions.**

**In the research, we found that the Chaboche model can be used as a material constitutive equation to accurately estimate the deformation of materials when repeatedly subjected to various stresses. This makes it possible to predict thermal deformation.**

## 1 Preface

Metal fatigue fracture is the main cause of mechanical destruction of products. There is a demand for improved technology to predict the appropriate product life. For example, copper is used as a conductor in electronic devices, and heats up when electricity is applied. This heat generates thermal strain, and loads are repeated as a heat cycle. As a result, the deformation of the conductor progresses, and it may eventually break.

Numerical analysis using the finite element method is sometimes used to predict deformation when subjected to repeated loads. The material constitutive equation that represents the characteristics of the material is actively studied in steel materials such as mild steel[(1)] and stainless steel[(2)]. However, there has been very little research on pure copper.

To address this issue, we experimentally obtained repeated stress-strain curves for copper materials under various conditions that are considered to be within the range of product applications. Furthermore, we selected a material constitutive equation that can be applied to numerical analysis. In this paper, we introduce a technology that can reproduce experimental results with numerical analysis and predict thermal deformation.

## 2 Basic Study of Material Constitutive Equation

Elasticity is when a material is loaded and then unloaded, and the material returns to its original state without permanent deformation. On the other hand, when the material does not return to its original state and permanent deformation remains, it is called plasticity. In addition, when a force is applied again in the same direction to a material that has once been plastically deformed, a phenomenon occurs in which plastic deformation does not occur unless the load exceeds the load that caused plastic deformation in the previous history. This phenomenon in which a material hardens is called work hardening. This is a factor in which the characteristics differ depending on the history of repeated loading. For this reason, the changes in the characteristics of a material when it is subjected to repeated loading must be experimentally performed under various conditions. With this background in mind, the following describes the characteristics of the pure copper used in this test and the test that meets the requirements.

### 2.1 Test Specimen

The material of the test specimen was oxygen-free copper (C1020H material) according to JIS H3140. Fig. 1 shows the shape and dimensions of the test specimen. The shape of the test specimen

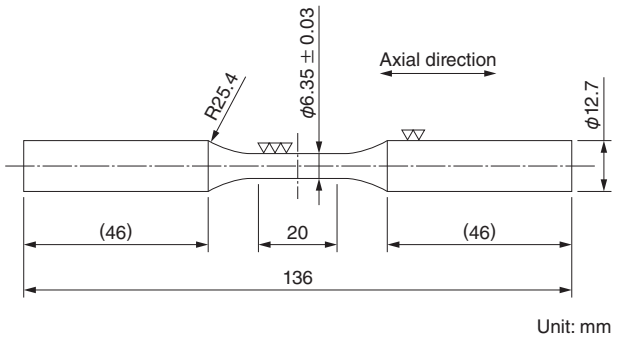



**Fig. 1** Shape and Dimensions of Test Specimen

The test specimen is a generally available round bar used to measure the strain in its center by exerting a stress on both ends. The entire body is made of pure copper.

was a round bar with a test section diameter of 6 mm, taking into consideration the prevention of buckling during compressive strain and heating.

### 2.2 Testing Machine

The testing machine used was a hydraulic servo fatigue testing machine manufactured by MTS Systems Corporation (MTS) in the United States. A high-temperature axial extensometer was used for strain measurement. In addition, high-frequency heating was used as the heating device.

### 2.3 Experimental Conditions

The repeated stress-strain curve was obtained by performing a strain control test assuming thermal strain. The amplitude $\varepsilon_a$ of the repeated strain was at 0.25%, 0.5%, and 0.75%, respectively. Temperature conditions were set at 20℃ (room temperature), 100℃, and 200℃, respectively. This is within the range in which the resin used to provide electrical insulation in the actual machine can be safely used.

## 3 Experimental Results

### 3.1 Basic Study of Material Constitutive Equation

First, the behavior of repeated stress-strain in pure copper was investigated. In this study, the material was annealed. The experiment was performed with an average strain $\varepsilon_m$ of 1.0% compression, 0.0%, and 1.0% tension, and the stress-strain curve was obtained by increasing the strain range by 0.1% for each cycle up to 2.0%. **Fig. 2** shows the repeated stress-strain curve for an average strain of 0.0%. **Fig. 3** shows the test results for each average strain. These are the ridges connecting the maximum stress and the minimum stress on the repeated stress-strain curve.

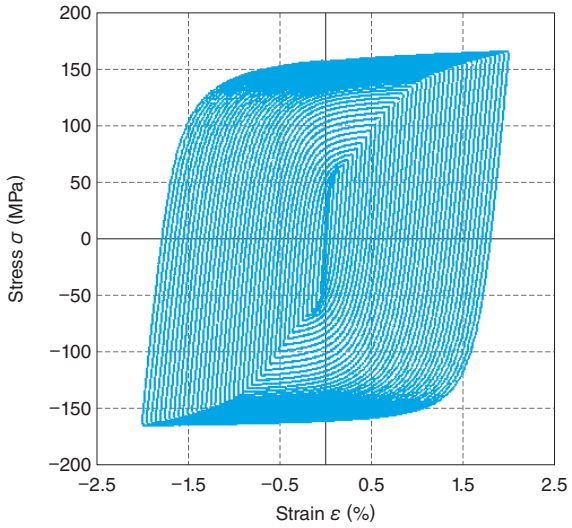



**Fig. 2** Repeated Stress-Strain Curves

Since the product is used repeatedly, the stress load is applied repeatedly also at the time of material testing. At that time, the history of stress and strain was recorded (increasing the strain value at the rate of 0.1% per cycle).

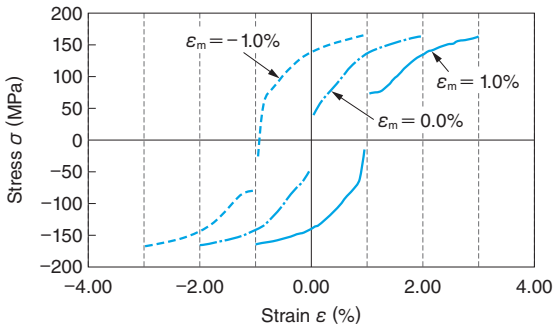



**Fig. 3** Cyclic Stress-Strain Curves (Average Strain Comparison)

Based on the history of stress and strain in **Fig. 2**, stress values in cases of maximum and minimum plus only strain values observed at that time are expressed by ridgelines. Three parallel lines show the cases when the strain is biased only to the pulling side ($\varepsilon_m = 1.0\%$), biased to the compression side ($\varepsilon_m = -1\%$), and no biasing ($\varepsilon_m = 0.0\%$), respectively. This experiment has simulated almost major operating conditions.

As a result, stress saturation was observed in the stress-strain curve for all average strains. On the other hand, as the average strain increased toward the tensile side, the macroscopic yield point tended to increase. From these results, it was found that the upper limit of stress on the stress-strain curve is finite. The isotropic hardening law, in which stress increases infinitely, is inappropriate. On the other hand, because of work hardening, kinematic hardening must be considered. For this reason, it

was found that a combined law of kinematic hardening and isotropic hardening with constraints is necessary.

### 3.2 Influence of Temperature and Strain Range

Fig. 4 shows the stress-strain curve[3] when strain is repeated 50 times. (a) shows the case where the temperature is 20℃ (room temperature), and the dotted line shows the case where the strain amplitude $\varepsilon_a$ is 0.25%, the dashed line shows the case where $\varepsilon_a$ is 0.5%, and the solid line shows the case where the amplitude is even larger, $\varepsilon_a$, of 0.75%. Comparing $\varepsilon_a$ of 0.25% and 0.5%, the absolute value of the maximum stress $\sigma_{max}$ increases. However, comparing $\varepsilon_a$ of 0.5% and 0.75%, it can be seen that $\sigma_{max}$ is almost the same and the stress is saturated. (b) shows the case where the temperature has risen to 100℃, and (c) shows the case where the temperature is 200℃. As the temperature rises, $\sigma_{max}$ decreases. In addition, when the temperature was 100℃ and 200℃, $\sigma_{max}$ was maximum at 0.25%, where $\varepsilon_a$ was small, and $\sigma_{max}$ was tended to decrease as $\varepsilon_a$ increased.

## 4 Material Constitutive Equations

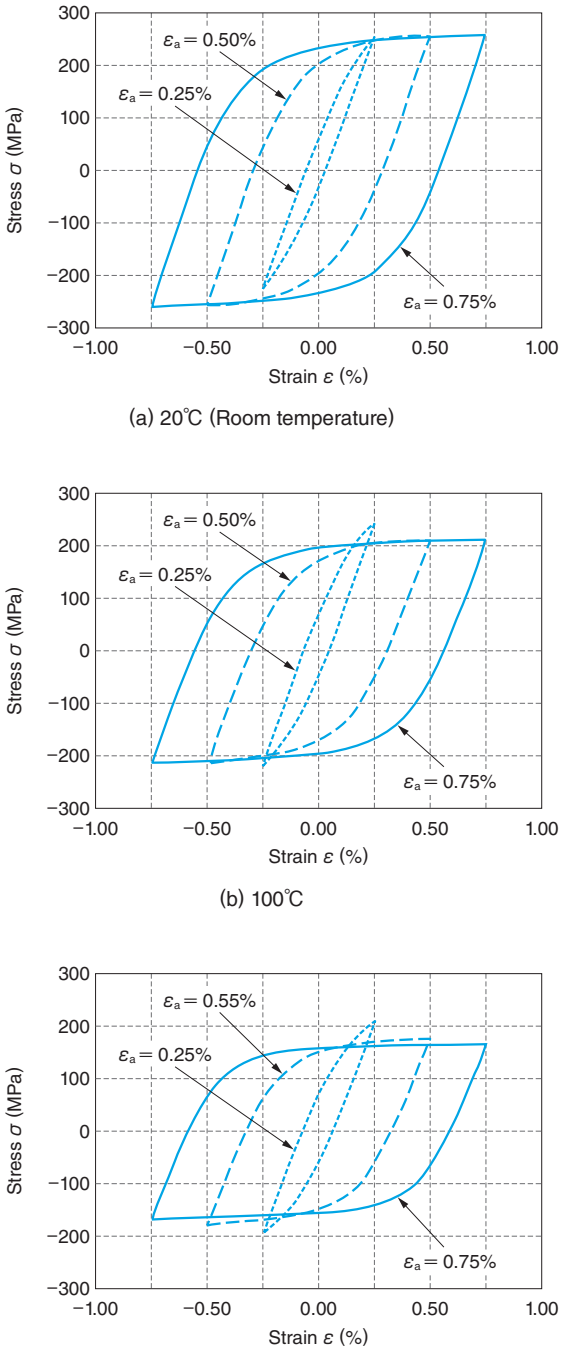
In the material constitutive equations of metallic materials, multilinear isotropic hardening and multilinear kinematic hardening are often used in numerical analysis. However, from the experimental results, there is an upper limit to the stress. In addition, work hardening and kinematic hardening must be considered. Accordingly, we tried to apply the Chaboche model [Expression (1)] which is currently being studied in steel[4].

$$\alpha = \sum_{i=1}^{n} \alpha_i \quad \dot{\alpha}_i = \frac{2}{3} C_i \dot{\varepsilon}^{pl} - \gamma_i \dot{\bar{\varepsilon}}^{pl} \alpha_i \cdots\cdots\cdots\cdots (1)$$

Where, $\alpha$ is back stress, $\varepsilon^{pl}$ is plastic strain, and $C_i$ and $\alpha_i$ are constants. The dot on these terms shows a differential of the quantity with time. The stress-strain curves obtained from testing were processed for the curve-fit by the aid of Expression (1). When Expression (1) is analyzed up to its 5th dimension, it was clarified that the result of experiments shown in Fig. 4 can be roughly reproduced.

## 5 Numerical Analysis Results

For the material constitutive equation obtained above, the general-purpose finite element analysis program Ansys 2021 R1 was used to calculate the stress-strain curve. The yield condition for the analysis was the Mises method, the full Newton-




(a) 20℃ (Room temperature)

(b) 100℃

(c) 200℃

**Fig. 4** Cyclic Stress-Strain Curves (Effect of Strain Range and Temperature)

Anticipating the use of conductors like coils, experiments are carried out under the circumstance of up to 200℃ to withstand temperatures for insulation materials such as resins. To secure versatility, the strain values are made to conform to three conditions, from a small strain to a large strain. It is realized that the stress-strain curves become flat in right and left directions and the amount of strain is increased when the temperature goes higher. Based on the relationship between stress and strain, we established a constitutive equation of materials with the use of a Chaboche model.

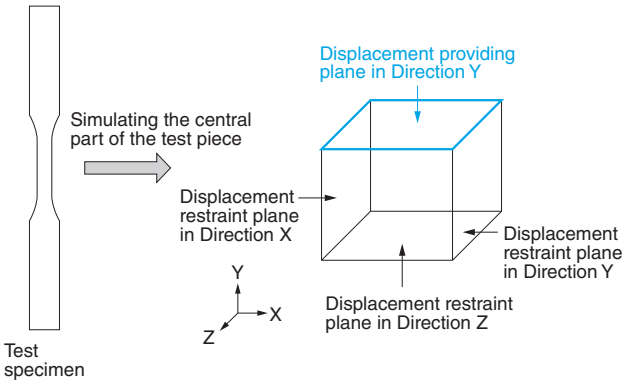


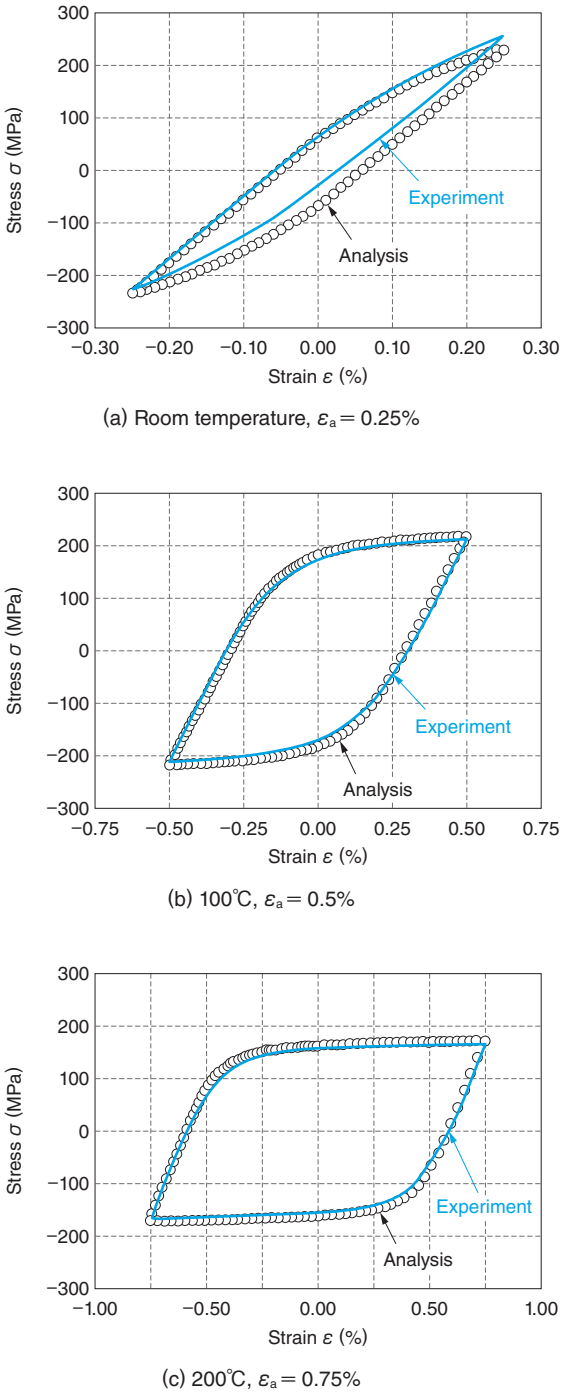

**Fig. 5** **Analysis Model and Boundary Conditions**

To reproduce the aforementioned test result utilizing the finite element method, the shape model we produced was a simple cubic body that simulates the central part of the test piece.

Raphson method was used as the solution method, and the default value was used for the convergence judgment. **Fig. 5** shows the analysis model and boundary conditions. The analysis model had 8 nodes and 1 element, with dimensions of 1 mm square. The boundary condition was set to a condition in which displacement was applied in the Y direction for each restraint surface in the XYZ directions. **Fig. 6** shows a comparison of the experimental and numerical analysis results[3]. The vertical axis is the uniaxial stress $\sigma$ in the tensile axis direction, and the horizontal axis is the total strain $\varepsilon$. The circles indicate the analysis results, and the solid lines indicate the experimental results. The data for strain amplitude $\varepsilon_a$ of 0.5% at 100℃ is used as the basis, and the data for $\varepsilon_a = 0.25\%$ at room temperature and $\varepsilon_a = 0.75\%$ at 200℃ are extracted and explained.

First, in the case of room temperature, the $\sigma_{max}$ obtained by the analysis is slightly smaller on the tensile side, but is almost the same on the compressive side. In the case of 200℃, the results are consistent with the experimental results. In addition, good reproducibility was confirmed at 100℃ as well as at 200℃. Creep strain superposition is considered at 200℃. However, a separate heat cycle test confirmed that the creep amount was small under these test conditions. In addition, in practical terms, there are cases where the final strain range cannot be predicted. Therefore, the constitutive equation obtained at $\varepsilon_a = 0.5\%$ at 100℃ was used to calculate the case of $\varepsilon_a = 0.75\%$ at 100℃ and compare it with the analysis.

**Fig. 7** shows a supplementary examination of the stress-strain curve. In this case, too, the stress-strain curve obtained by analysis closely matched the experimental results.




(a) Room temperature, $\varepsilon_a = 0.25\%$

(b) 100℃, $\varepsilon_a = 0.5\%$

(c) 200℃, $\varepsilon_a = 0.75\%$

**Fig. 6** **Comparison of Experimental and Numerical Analysis Results**

According to the nine conditions shown in **Fig. 4**, numerical analysis was carried out with the use of a Chaboche model for the constitutional equation of materials. Based on the obtained analytical data, the result of comparison is shown between the result of experiments on three typical patterns and that of numerical analysis. When strain is small, the strain value from analysis becomes excessive, thus resulting in a fact that evaluation stays on the safe side. When strain is increased up to a certain level, on the other hand, the result of coincidence is obtained. Regarding the other six patterns not shown in this figure, similar trends were observed.

These results suggest that it is possible to pre-

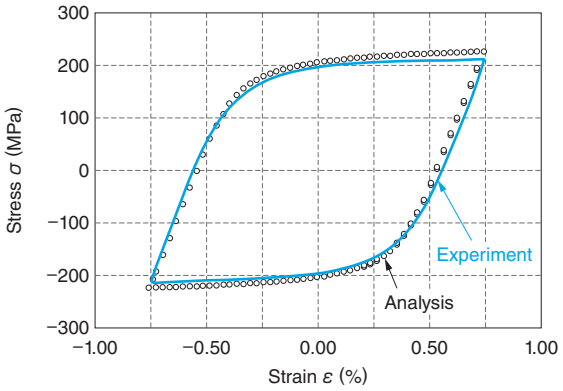



**Fig. 7** **Supplementary Examination of Stress-Strain Curve**

To examine the adaptability of experiments deviating from the empirical conditions shown in **Fig. 4**, the constitutive equation obtained from the conditions of $\varepsilon_a = 0.5\%$ at 100℃ was adopted and numerical analysis was attempted under the conditions of $\varepsilon_a = 0.75\%$ at 100℃. The tendency was such that the strain value gets slightly smaller in the case of numerical analysis, but we realized that a certain level of prediction is possible.

dict the amount of thermal deformation and examine low-cycle fatigue using elastic-plastic analysis, even for pure copper, which is considered to have a large nonlinearity.

## 6 Postscript

This time, a material constitutive equation that can be used for numerical analysis of pure copper had been selected, making it possible to predict thermal deformation.

Going forward, we will continue to promote the development of even more accurate life prediction technology in order to contribute to reducing environmental impact and improving product competitiveness.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